# Annual Default Rates are Probably Less Than Long-Run Average Annual Default Rates

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**Abstract**

Banks using either the Foundation or Advanced option of the Internal Ratings Based approach to credit risk under Basel II must estimate long-run annual average default probabilities for buckets of homogeneous assets. The one-factor model underlying the capital calculations in Basel II has implications for the distribution of average (across assets) default rates over time. One of these implications is that the average default rate in any period is probably smaller than the overall average default rate (over time and assets). The lesson for practioners is that the short-term default experience of new, very safe assets is likely to underpredict the true long-run default rate for these assets.

## 1 Introduction

The Basel II (B2) capital requirements - see Basel Committee on Banking Supervision (2006) - are based on a one-factor model due to Gordy (2000) that acommodates systematic temporal variation in asset values and hence in default probabilities. This model can

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be used as the basis of a model that allows temporal variation in the default probabilities, and hence correlated defaults within years. The value of the ith asset in time t is

$$v_{it} = \rho^{1/2} x_t + (1-\rho)^{1/2} \epsilon_{it}$$

where $\epsilon_{it}$ is the time and asset specific shock and $x_t$ is the common time shock, inducing correlation $\rho$ across asset values within a period. The random variables are assumed to be standard normal and independent. A mean of zero is attainable through translation without loss of generality since we are only interested in default probabilities. Suppose default occurs if $v_{it} < d$, a default threshold value. The overall or marginal default rate apparently required by B2 is $\theta = \Phi(d)$. However, in each period the default rate depends on the realization of the systematic factor $x_t$; denote this $\theta_t$. The model implies a distribution for $\theta_t$. Specifically, the distribution of $v_{it}$ conditional on $x_t$ is $N(\rho^{1/2} x_t, 1-\rho)$.

Hence the period $t$ default probability is

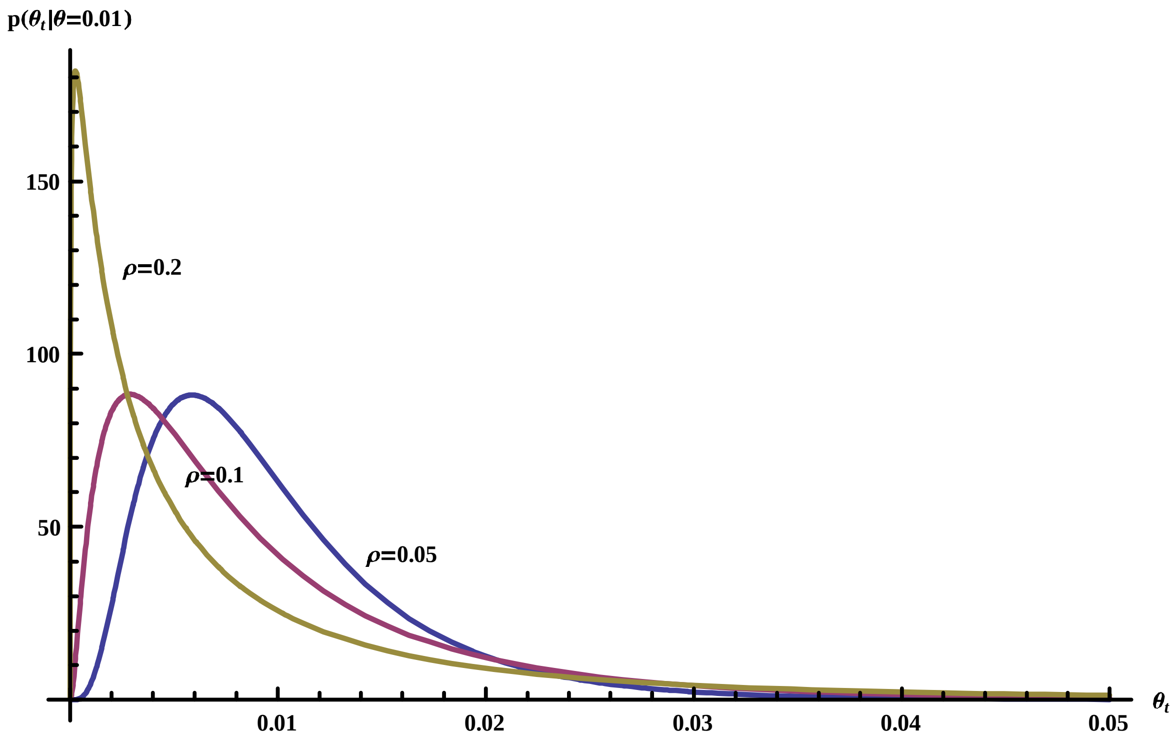
$$\theta_t = \Phi[(d - \rho^{1/2} x_t)/(1-\rho)^{1/2}]$$

Thus for $\rho \neq 0$ there is random variation in the default probability over time. The distribution is given by

$$\begin{aligned} \Pr(\theta_t \leq A) &= \Pr(\Phi[(d - \rho^{1/2} x_t)/(1-\rho)^{1/2}] \leq A) \\ &= \Phi[((1-\rho)^{1/2} \Phi^{-1}[A] - \Phi^{-1}[\theta])/\rho^{1/2}] \end{aligned}$$

using the standard normal distribution of $x_t$ and $\theta = \Phi(d)$. This is known as the Vasicek distribution. Differentiating gives the density $p(\theta_t|\theta, \rho)$. The parameters are $\theta$, the marginal or long-run mean default probability and the asset correlation $\rho$. Values for $\rho$ are in fact prescribed in Basel Committee on Banking Supervision (2006) for different asset classes. There is very little data evidence on $\rho$ so we defer to the B2 formulas and set $\rho$ to a prescribed value (0.20), but give results to show sensitivity to a range of values of

**Figure 1: Densities $p(\theta_t|\theta=0.01)$ for different $\rho$**



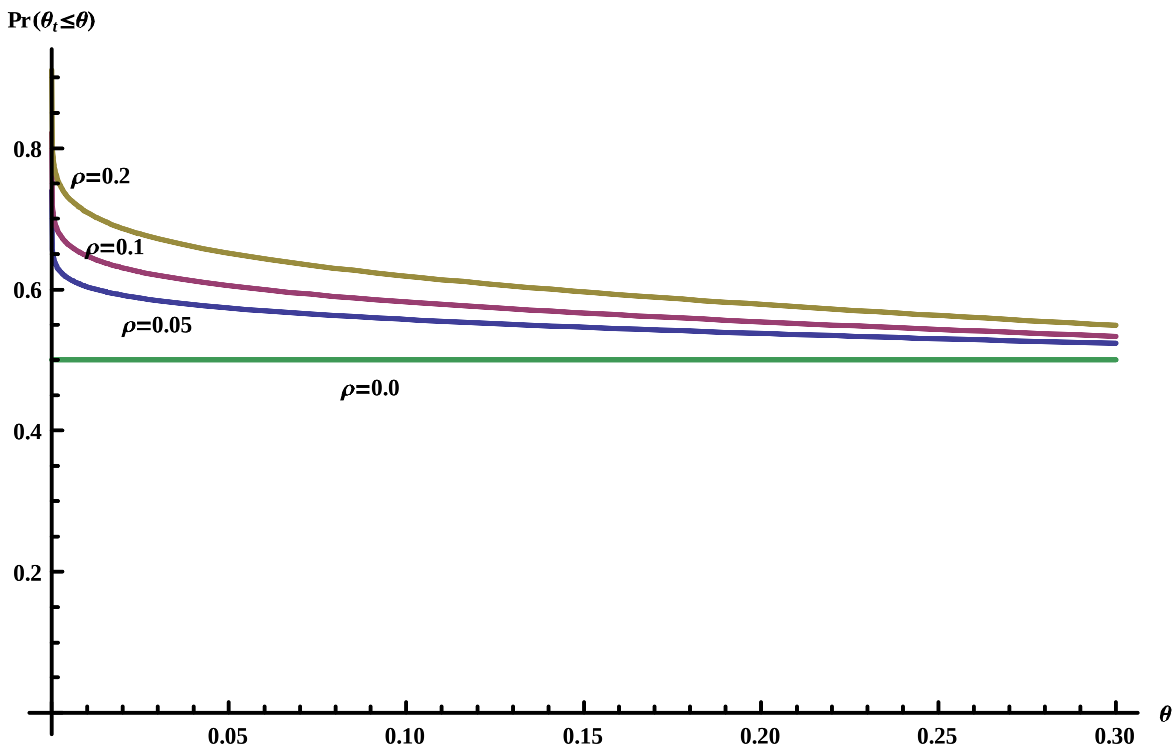

the asset correlation. To provide some background, Figure 1 plots the density $p(\theta_t|\theta, \rho)$ for $\theta = 0.01$ and $\rho = \{0.05, 0.1, 0.2\}$.

All of these densities have mean approximately equal to 0.01 but it is clear that the mass below the mean depends crucially on the correlation $\rho$.

## 2 Implications

We claim that default rates annually are more likely to be less than the long run average than to be above when defaults are correlated. To illustrate the claim, let us use a mid-portfolio value for the long-run default rate $\theta$, say $\theta = 0.01$ (100 basis points). With the asset correlation set at 0.2, the probability that $\theta_t$, a draw from the density $p(\theta_t|\theta, \rho)$, is less than 0.01, the long-run default rate, is 0.709, over two thirds. Thus, we are more than twice as likely to see a realized default rate in any given year that is below the long-run rate than one above the long-run rate. How does this occur? Not because the location

**Figure 2: Probabilities $\Pr(\theta_t \leq \theta)$ for different $\rho$**




is incorrect, the mean of the distribution of $\theta_t$ is almost exactly 0.01, the long-run rate. Rather it is because lower than expected rates, occurring about 71% of the time, are expected to be 0.0034, or about 1/3 of the long run value, while higher than expected rates, when they occur, are expected to be 0.026, 2 1/2 times the long-run value.

Having set the stage with an example, we turn to a systematic description of the relationship between the long-run default rate and the probability that an annual realized default rate will be below the long-run rate. Figure 2 graphs the probability that the realization is less than the mean against the mean (long-run rate).

For low values of the default rate, correlation has stronger implications that the observed defaults will be fewer than expected. Thus for very safe highly correlated assets the realized default rate in a short period is likely to be substantially lower than the true long-run average default rate. The effect is reversed as the long-run rate crosses 0.5 (not a relevant default rate, one hopes). The graph is drawn for $\rho = (0.05, 0.1, 0.2)$, perhaps covering the relevant range of asset correlations. For $\rho = 0$ the annual default rate is equal to the long-run average for any value of the default rate.

## 3 Conclusion

Average default rates in any year are likely to be less than the long-run average when defaults are correlated. This fact provides substantial support for the B2 insistence on data covering a full business cycle for estimating the long-run average default. It raises special questions for very new assets. Practitioners should be wary of short data sets especially on new, very safe assets. When default rates are correlated, the default rates experienced over the first year or two are likely to substantially underestimate the long-run average default rate. When the asset correlation is 0.2, a value suggested by B2 when default rates are near 0.01, the understatement in a year of experience might be by a factor of 1/3. Perhaps it would be useful to rely on expert judgment about these assets, at least until adequate data series are observed. See Kiefer (2007). Is this observation relevant for recent experience with new asset types?

## References

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